

MATH 141: Quiz 2

Name: key

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

1. Rationalize and completely simplify:

$$\frac{A - B}{\sqrt{x} - \sqrt{x+h}} \cdot \frac{A + B}{\sqrt{x} + \sqrt{x+h}}$$

$$= \frac{\overbrace{x}^{A^2} - \overbrace{(x+h)}^{B^2}}{h(\sqrt{x} + \sqrt{x+h})} = \frac{x - x - h}{h(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-h}{h(\sqrt{x} + \sqrt{x+h})} \stackrel{\text{frac law 5}}{=} \boxed{\frac{-1}{\sqrt{x} + \sqrt{x+h}}}$$

2. Suppose $f(x) = x - x^2$. Evaluate and completely simplify/factor the following:

(a) $f(-1) = (-1) - (-1)^2 = -1 - 1 = \boxed{-2}$

(b) $f(x+h) - f(x)$

$$= \overbrace{(x+h) - (x+h)^2}^{f(x+h)} - \overbrace{(x - x^2)}^{f(x)}$$

$$= x+h - (x^2 + 2xh + h^2) - x + x^2$$

$$= x+h - x^2 - 2xh - h^2 - x + x^2$$

$$= h - 2xh - h^2 = h \cdot (1 - 2xh - h)$$

3. Given the problem

$$\frac{x^{-1} + y^{-1}}{2}$$

(a) What type of fraction structure is this called?

Compound fraction

(b) Now fully simplify this expression.

for a compound fraction, this means no nested fractions.

$$\frac{x^{-1} + y^{-1}}{2} \stackrel{\substack{\text{def} \\ \text{of } a^{-n}}}{=} \frac{\frac{1}{x} + \frac{1}{y}}{2} \stackrel{\substack{\text{treat numerator as} \\ \text{subproblem}}}{=} \frac{\frac{y}{y} \cdot \frac{1}{x} + \frac{1}{y} \cdot \frac{x}{x}}{2}$$

$$\stackrel{\text{frac law (1)}}{=} \frac{\frac{y}{xy} + \frac{x}{xy}}{2} \stackrel{\text{frac law (2)}}{=} \frac{\frac{x+y}{xy}}{2}$$

$$\stackrel{\text{frac law (2)}}{=} \frac{x+y}{xy} \cdot \frac{1}{2} = \boxed{\frac{x+y}{2xy}}$$

4. Suppose we have a function and

$$\lim_{x \rightarrow 3} f(x) = 4$$

is true. What must be true for the one-sided limits?

The left-hand and right-hand limits must be equal.